Mathematics teaching and learning as an ethical event

Insegnamento e apprendimento della matematica come un evento etico

Enseñanza y aprendizaje de las matemáticas como un evento ético[1]

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Abstract. *The main claim of this article is that mathematics teaching and learning is unavoidably an ethical event. This claim is based on the idea that teaching and learning rests on (1) relations between individuals (e.g., relations of power, relations of solidarity) and (2) the legitimation of particular forms of knowledge and knowing. From an educational viewpoint, the question that arises in this context is the kind of ethics that mathematics pedagogies could strive to nurture. The answer, it is argued, depends on the educational theory or theories to which one resorts to understand teaching and learning. The article ends with a sketch of a communitarian oriented relational ethics as articulated in the theory of objectification—a communitarian ethics whose practice features responsibility, commitment, and care.*

Keywords: ethics, theory of objectification, Vygotsky, Spinoza, Lévinas, Hegel.

Sunto. *L'affermazione principale di questo articolo è che l'insegnamento-apprendimento della matematica costituisce inevitabilmente un evento etico. Questa affermazione si fonda sull'idea che l'insegnamento e l'apprendimento si basano su (1) relazioni tra individui (per esempio, relazioni di potere, relazioni di solidarietà) e (2) sulla legittimazione di particolari forme di conoscenza e sapere. Da un punto di vista educativo, la domanda che ci si pone in questo contesto è il tipo di etica che le pedagogie matematiche potrebbero sforzarsi di coltivare. La risposta, si sostiene, dipende dalla teoria o dalle teorie educative a cui si ricorre per comprendere l'insegnamento e l'apprendimento. L'articolo termina con uno schizzo di un'etica comunitaria orientata all'etica relazionale come si articola nella teoria dell'oggettivazione – un'etica comunitaria la cui pratica caratterizza responsabilità, impegno e cura.*

Parole chiave: etica, teoria dell'oggettivazione, Vygotskij, Spinoza, Lévinas, Hegel.

Resumen. *La afirmación principal de este artículo es que la enseñanza-aprendizaje*

[1] Invited article/Articolo invitato/artículo invitado.

de la matemática constituye inevitablemente un evento ético. Esta afirmación se apoya en la idea de que la enseñanza y el aprendizaje se basan en (1) las relaciones entre individuos (por ejemplo, las relaciones de poder, las relaciones de solidaridad) y (2) en la legitimación de formas particulares de conocimiento y saber. Desde un punto de vista educativo, la pregunta que surge en este contexto es el tipo de ética que las pedagogías matemáticas podrían esforzarse por cultivar. La respuesta, se argumenta, depende de la teoría o teorías educativas utilizadas para entender la enseñanza y el aprendizaje. El artículo termina con un esbozo de una ética comunitaria orientada a la ética relacional tal como se articula en la teoría de la objetivación – una ética comunitaria cuya práctica caracteriza la responsabilidad, el compromiso y el cuidado.

Palabras claves: ética, teoría de la objetivación, Vygotsky, Spinoza, Lévinas, Hegel.

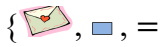
1. Introduction

The first question that might arise is the following: What does ethics have to do with mathematics education? Let me present a short twofold answer.

First answer

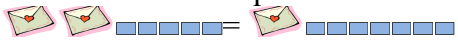
Teaching-learning mathematics cannot avoid facing the question of the *legitimation* of particular forms of knowledge and knowing that arise in the classroom. Classroom discussions usually lead to *conflicting views* about what counts as mathematically valid and authentic.

Here is an example. In a Grade 5 class (10-11-year-old children), the students were invited to write a text for a student from another class explaining how to solve linear equations. The students had been using an iconic semiotic system (ISS) to write and solve simple equations:



In the ISS, the small rectangles represented cards; the envelope represented the unknown (as each envelope contained the same unknown number of cards). The problems with which the students had been dealing so far involved two individuals (e.g., Claudine and Sylvain) who each had a known number of cards and one or more envelopes. The individuals' total number of cards was the same.[2]

Some students suggested a text based on a concrete example (see Figure 1, left, where the text revolves around the equation:



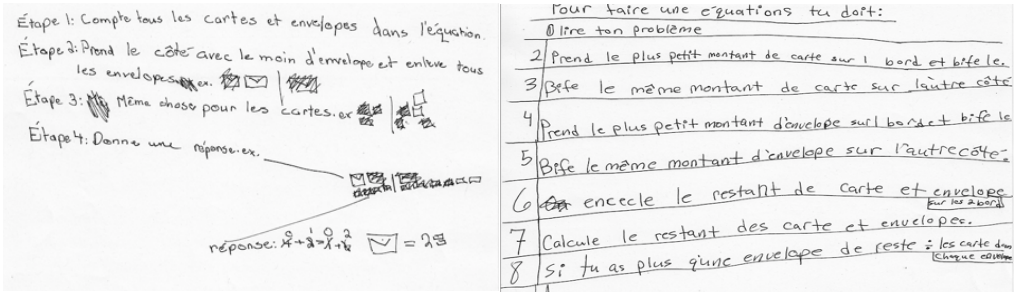
where the two sides of the equations were divided by a vertical line). But other students suggested a text without any concrete example (see Figure 1, right).

Which text is better? And if you were teacher, what would you say to the students?

[2] For an example of the classroom equation activities, see, e.g., Radford, Demers, and Miranda (2009).

Figure 1

Two mathematical texts. Which one is better?



Since there are plenty of ways in which to think mathematically, taking sides or suggesting something else involves a question of power, and since there is a question of power there is also a question of subjecting people to a *particular* image of mathematics.

Second answer

My second answer is not about positions taken on questions of mathematical legitimacy but about *relations* between people.

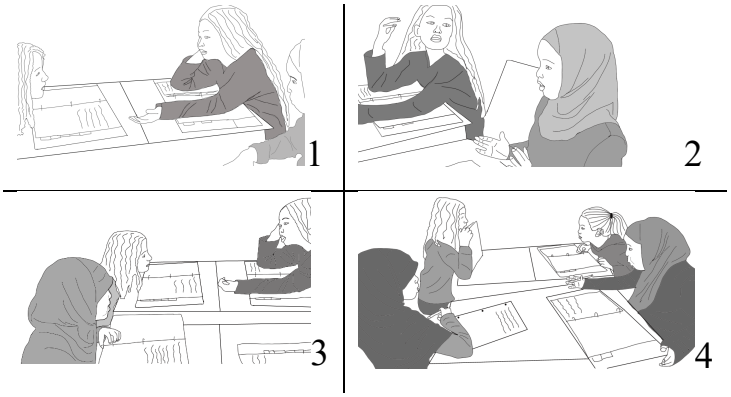
Teaching-learning mathematics in the school involves *interaction* between people. Teaching-learning is based on relations with others, and these relations involve *necessarily* an ethical dimension: In classroom interaction we have, for instance,

- relations of power and subjection,
- relations of *authority* and *obedience*, and
- relations of *solidarity* and inclusiveness.

Here is an example. In a Grade 4 classroom (9-10-year-old children), the students were working in small groups of three or four trying to solve geometric problems. The first problem revolved around the classical question of whether squares are rectangles. The pictures in Figure 2 provide a sample of *body positions* of the interaction. In Picture 1, Laura is talking to Sandra: "Yes, but they have all four sides." In Picture 2, Mirna tries to contribute to the group and says: "The squares have same umm … the same edges …" Laura turns to look at Mirna for a short period of time; then, turns back to look at Sandra to continue their discussion. In Picture 3, Híria (front left) tries unsuccessfully to get Sandra's and Laura's attention and says: "The squares have parallel faces … because there is … Look!" In Picture 4, after recurring attempts to be heard, Mirna expresses her frustration and utters an anguished "Ahhhhh!!!"

Figure 2

A group of Grade 4 students dealing with a geometry problem



We are here in the presence of an ethics of exclusion. The practice of an ethics of exclusion impedes a genuine collective engagement in mathematics and raises an invisible, yet important, wall between *us* and *them*.

The previous examples illustrate the main idea of this article: teaching-learning is unavoidably an *ethical* event—and this is so regardless of the pedagogical model that underpins it.

I would like to go a step further and contend that ethics is not only omnipresent in mathematics teaching and learning but is also a crucial component of it. Indeed, first, ethics shapes the manners in which teachers and students *engage* and *assume* (or not) certain *responsibilities* in the mathematics classroom; consequently, ethics shapes how teachers and students come to understand mathematics and conceive of themselves as practicians of mathematics. Second, ethics shapes the *students' and teachers' relationships with others*—for instance, in the various manners by which the students voice (or not) their values and understandings, and how their voice is heard (or not). In this sense, ethics affects how teachers and students assert themselves as mathematical *subjectivities*.

The importance of ethics in an encompassing account of learning leads us to the question of the kind of ethics that we could nurture in the classroom. In this article I sketch a conception of ethics as articulated in the theory of objectification—a teaching and learning theory inspired by dialectical materialism and Vygotsky's school of thought (Radford, 2019a, 2021a). In this ethics we move away from contractualist conceptions of ethics, such as Thomas Hobbes's, where individuals give and take to preserve the social order and their order in it (see, e.g., Hobbes, 1841; analysis in Radford, 2021a). We also move away from the idea of ethics as something based on principles that can be universally applicable, as in Immanuel Kant's rational morality (see, e.g., Kant, 2006; analysis in Radford 2021a). We rather resort to a concept of ethics as something that is materialized as we engage in the world (Roth,

2013, 2017) and that is intrinsically *ambiguous* and *context sensitive* (Bauman, 1993; Boyland, 2016): a *communitarian relational ethics* whose practice features responsibility, solidarity, and care.

Before I go into the account of ethics in the theory of objectification, in the next section I deal briefly with what *makes ethics possible* in the first place.

2. The possibility of ethics: Free will

The starting point is that, in a very fundamental ontological sense, we are beings of choice. We are beings of free will. This means that, every moment, our deeds can take one direction or another direction. If we were compelled to always carry out certain actions, like machines, ethics would not arise. It is because we can choose our actions, it is because of our free will, that ethics arises.

It is precisely not in the cognitive sphere but in the *scope* of the exercise of free will that Vygotsky found the most distinguishing feature of humans within the broad spectrum of natural living beings (del Rio & Alvarez, 1995). Free will manifests itself in the "struggle of motives" (Vygotsky, 1997, p. 167). Individuals, Vygotsky says, "never sense themselves as free to act on their own as when they confront several possibilities and actions simultaneously and, as if in a free act of will, make a choice between them" (p. 168; citation grammatically adjusted). This is why ethics "will always be that which is associated with the free choice of social forms of behavior" (Vygotsky, 1997, p. 226). Commenting on Spinoza's view, Vygotsky goes on to say that, according to the Dutch philosopher, "if a person runs away from something on the grounds that it is bad, he is acting like a slave. Only that person is free, in Spinoza's view, who runs away from something because something else is better" (Vygotsky, 1997, p. 226).

Of course, Vygotsky does not see free will as something unbounded. On the contrary, for him, humans will always stand within the confines of the social and political order. So, the discursive and non-discursive practices and technologies that underpin and shape our world do constrain, afford, and promote certain forms of action and choice. However, these practices do not impede us from thinking and acting *differently*. There are always possibilities to *interrupt* the quotidian train of our actions and thinking. It is at this point that Scott's (1990) *The Question of Ethics* as a question of *interruption* of habits and values acquires its whole sense. For Scott, *The Question of Ethics* indicates our capacity to bring forward "an interruption in which the definitive values that govern thought and everyday action lose their power and authority" (p. 4). This interruption makes sense precisely because of our cultural-historical *agentic* nature (i.e., our power to sublate and surpass the cultural and historical possibilities on which we draw when we re-act to, and re-enact, the world).

This agentic nature does not derive from a privileged position that makes the subject the *source* of meaning and intentionality, as articulated by the rationalist and empiricist trends of the Enlightenment. Nor does the agentic nature remain confined to "the effect of the subject-positions articulated in discourse" (Atkins, 2005, p. 252)—a theoretical conception that, as Nealon (1998) argues, rests on an idea of self as *lack*: lack of wholeness or plenitude, subjected forever to the limits of the given structures and confining discourses.

Historically speaking, feminist, multiculturalist, dialectical materialist thinkers and scholars in related fields have countered these two conceptions of agency mentioned above:

(1) self as *source*—i.e., as the origin of meaning, knowledge and intentionality, the self that is featured in Piagetian and constructivists accounts and that gets caught in a solipsist world unable "to look behind its back to see what unacknowledged truth its own activity is reflecting" (Russon, 2004, p. 187), unable thus to recognize that the alienating confinement it finds itself in is the product of a range of social determinations—and

(2) self as *lack*—i.e., as a being that, as in the case of self as source although without claiming to be the origin of cognitive life, is unable to challenge and transform the cultural-historical structures that confine it.

These scholars have countered these conceptions of self and agency by offering a concept of the subject as *excess*. In feminist theory the subject is conceived of as having the power "to reflect on the social discourse and challenge its determinations" (Alcoff, 1988, p. 417). Following a Hegelian thread, for Butler (1999), the subversion of the subject is possible because all acts of signification not only restrict the subject's actions but are, at the same time, in their enactment, always located within the possibility of a variation in the "alternative domains of cultural intelligibility" (p. 185). Dialectical materialist thinkers (e.g., Fischbach, 2014; Macherey, 2008), have drawn on Marx's (1998) work where the subject is featured as one that, while being produced by its circumstances, has, inversely, the power to transform those circumstances. Following Marx, what dialectical materialists add to the agentic conception of the self is that subversion is accomplished in *praxis*, *with others* (Freire, 1998). For consciousness is not only a refraction of reality. Most importantly, consciousness, along its varying layers of depth, is a concrete relation that, given our biography and cultural background, *propels us* towards the world and leads us to act on/in it and *transform* it (Clot, 2015). There is still hope, then, that, in our Grade 4 example, Mirna and her twin sister, Híria, will be heard. However, for this to occur, there must be a transformation of circumstances. The classroom culture must be transformed. This transformation requires a new *praxis*, a classroom praxis, out of which a

new form of social consciousness can emerge.[3]

3. Ethics in the theory of objectification

In the introduction I mentioned that one of the main features of ethics as conceived of in the theory of objectification is that it is intrinsically *ambiguous*. To be ambiguous means that ethics is not something contractual (like teachers do this, students do that), nor is it something that works on the basis of rules and abstract principles (like *do your homework!*). To say that ethics is intrinsically ambiguous means that our acts and relations to others do not have *one* obvious meaning. They are context sensitive. So, ethics is a context-sensitive dynamic and open-ended relational stance that is continuously materialized and assessed as teachers and students engage in mathematical activity.

The idea of ethics as ambiguous and context sensitive might be better understood if we bear in mind that ethics in the theory of objectification follows a Vygotskian-Spinozist line of monist thinking that relates ethics to *consciousness* and *emotions*.

Consciousness comes into the scope of ethics as that which can help us understand the meaning of our deeds, to reflect on them, and to imagine *new* courses of action. Consciousness is at the heart of what Spinoza called the body's *power* of acting in the world (*agendi potentia*).

Since Spinoza thought of individuals as *bodily sentient beings*, the individuals' deeds and thoughts have unavoidably an *emotional* dimension. Thus, "every man [*sic*], according to his emotions (*ex suo affectu*), judges a thing to be good or bad, useful or useless" (Spinoza, 1989, p. 153). More generally, "having an idea is at the same time being in an affective state" (Bijlsma, 2014, p. 7). But Spinoza's ethics stresses another fundamental point: we, humans, are primarily continuously affected by our contexts; we are inter-reliant in the ways we come to know and feel toward each other. He offers a picture of human beings "as fundamentally interdependent beings, whose passions and opinions are continuously aroused, reinforced and transformed by those of their fellows" (Bijlsma, 2014, p. 10).

The theory of objectification also follows Lévinas's (1982) *relational* ethics that radicalizes previous ethical systems in acknowledging that our actions and deeds are always modulated by the presence of the *Other*—a presence that comes to us in a sentient and fleshy manner: through the *proximity of our bodies*. In this proximity our conceptual epistemological categories and mechanisms are put on hold, and we encounter the Other as is. "The Other is appreciated precisely *as* Other, in her radical alterity and

[3] The idea of the transformative praxis was quickly sketched in Thesis 3 of Marx's *Theses on Feuerbach.* For an enlightening analysis see Macheray (2008); see also Fischbach (2015).

irreducible singularity, only when thought renounces its totalitarian hubris and learns to think of the Other on her own terms . . . 'beyond essence'" (Min, 1998, pp. 573–574; emphasis in the original). In giving an ontological primacy to the Other, Lévinas's ethics removes the self from the privileged seat with which it has been traditionally endowed and from where it has conceived of itself as a constituting consciousness and the principle of the ethical relationship. In Lévinas's ethical account, it is not from the self and its deeds that ethics appears. Stripped of its imperialism, the self appears conceived of as the *result* of an ethical relationship.

Drawing on those insights, ethics in the theory of objectification is understood as the *form* of *alterity* (the *form* of our relationship to the Other). The term "form" is used here in its dialectical materialist sense: just as there are forms of property and forms of society, there are forms of human relations. Forms are the very expressions of the social, cultural, and historical backgrounds that frame them.

In conceiving of ethics as the form of alterity, the focus turns not to moral precepts but rather to the fluid and content-dependent relationships between subjects as they appear in the immediacy and banality of everyday life. In this view, ethics is continuously *materialized* in *praxis* out of a myriad of possibilities, for, in this view, the individual appears as "full of unrealized possibilities every minute" (Vygotski, 2003, p. 76). If we come back to Figure 2, we see that Mirna's teammates could have opted for other actions. The materialization of the students' *actions* reflects their understanding of the context (*consciousness*) and the manner in which the context is lived through their unfolding collective affective experience (*emotions*), as well as the relational stances (*ethics*) they (consciously or not) adopt towards each other. Through the previous classroom examples, we see that embodied, emotional, contextual, cultural-historical action is related to ethical postures that students assume and show in practice.

In the introduction I claimed that all educational theories resort to a certain ethics, explicitly or implicitly. This is so because *the ethics of a theory reflects on the realm of social relations, the manners in which the theory expects teaching and learning to occur*. It is, indeed, from this expectation about learning that roles and relations become assigned to the participants.

Let us consider two examples.

Think of the theory of transmissive instruction. Learning is conceived of as the assimilation, through practice and repetition, of knowledge that the teacher possesses. The theory positions teachers as knowers and the students as lacking knowledge. The ethics of the transmissive instruction, manifesting itself in the form of alterity that it promotes, reflects, but also operationalizes, the ensuing alienating relations of power and subjection, relations that are thematized along the lines of superior/inferior, potent/impotent, knower/ignorant, authority/vassal (Radford & Lasprilla Herrera, 2020).

Now think of constructivism. In contradistinction to theory of transmissive instruction, constructivism, is based on an ethics that stresses the freedom of the student: since knowledge is conceived of as what results from the autonomous deeds of the student, and learning is the very process of the student's subjective construction of knowledge, teachers and students are positioned otherwise: the student's freedom and autonomy configure the constructivist's ethical space (Radford, 2012).

In the theory of objectification, learning is conceived of differently from constructivist and transmissive instruction theories: learning is seen as a *collective* and *truly social* embodied and material process through which students critically encounter culturally and historically constituted ways of thinking mathematically. This encounter happens in what the theory terms *joint labour* (Radford, 2020).

Joint labour is a sensuous, practical, material *activity*—activity understood as driven by *collective* concerns. The German and Russian languages have a specific term for this type of activity: *Tätigkeit* and *deyatel'nost',* respectively. Activity in this sense is opposed to activity as being merely busy with something (as in watching TV). Again, the German and Russian languages have a specific term for this other type of activity: Aktivität or aktivnost'. Unfortunately, in the translation into English (and several other languages), the distinction is lost and both types of activity are rendered as *activity*. In the case of the theory of transmissive instruction, classroom activity is not oriented towards the satisfaction of collective needs. This activity corresponds hence to Aktivität or aktivnost'. In joint labour, by contrast, students and teachers work hand in hand to *produce* something *together*, what Hegel termed "a common work," in our case, *mathematics*. It is this sense of labouring together (as opposed to simply interacting or exchanging with others) that makes joint labour a truly social activity and learning a collective process. Classroom research has shown us, however, that for learning to become a truly collective process, radical changes in the classroom culture might need to occur. Often, drawing on experiences shaped by transmissive instruction, the students conceptualize the teacher as the possessor of knowledge and power, and conceptualize themselves as submissive to the teacher and her knowledge, even when the teacher tries to conceptualize herself differently and encourages the students to learn collectively and organizes the classroom into small groups. Often, the students configure small, enclosed groups and erect aggressive or exclusive antagonistic barriers between their group and other groups—they resort to what we may term a *clique ethics* or *gang ethics* (Radford & Lasprilla Herrera, 2020), which is also what we see in the example from the Grade 4 classroom briefly mentioned in the Introduction.

Of course, the educational problem around ethics is not the imposition of new social forms of conduct. "It is not obedience to someone or obedience to

something, but the free adoption of those patterns of behavior which will vouchsafe the consonance of all of behavior" (Vygotsky, 1997, p. 233). The educational problem around ethics becomes the problem of the creation of classroom conditions for new ethical relations (new *forms* of alterity) to emerge and to be collectively pondered and discussed against the always contested background of culture and history.

Now, the new ethical relations that our pedagogies could strive to nurture need to be congruent with the theoretical tenets of the theory. In the case of the theory of objectification, the educational problem becomes the problem of the creation of classroom conditions that, moving within a critical space of engagement, inclusiveness, debate, and respect, would be conducive to a *collective* practice of mathematics.

Of course, the teacher does not do the same things as the students. Yet, the teacher alone cannot produce mathematics, for this production, according to the tenets of the theory, is sought to be a *collective* production. The teacher finds herself in the same position as the conductor of an orchestra who might know a musical piece from A to Z but is not able to produce music by herself. Like the orchestra conductor who needs her musicians, the teacher *needs* the students (Radford, 2019b). This need is not merely practical. It is ontological, for in producing mathematics the teacher produces herself: she produces herself in her dealings with the students—and vice versa. This is why teachers and students co-produce themselves. The teacher is a difference between the equal, and an equal among the different. And so are the students.

In this view of learning as a collective process, the teacher is dethroned from the traditional role that sociocultural theories and other theories bestow upon her: that of a mediator or a scaffolder or a helper or a coach. This patriarchal role is replaced with one in which the teacher struggles, suffers, and finds enjoyment with the students in making mathematics a sensible common work.

So, what are the new ethical relations that we strive to nurture in the theory of objectification? We focus on a classroom mathematics practice featuring what we call a *communitarian ethics* based on responsibility, commitment, and care. It is here where we resort to the construct of "voice"—not voice in a linguistic sense exactly; rather we resort to voice as something that brings in the postmodern notion of *difference* and the *primacy of the political* (Giroux, 2005). Finding one's voice or having a voice is "moving from silence into speech," it is "a gesture of defiance that heals, that makes new life and new growth possible" (Hooks, 2015, p. 29), something that "assumes a primacy in talk, discourse, writing, and action" (p. 33). Coming back to our Figure 1, when Mirna utters an anguished "Ahhhhh!!!" and moves her right hand towards the two other girls who are not listening, she is moving into speech to express her frustration about not being counted and heard. Her voice (which is much more than what she discursively *says*, as it also says things in her body

posture, facial expression, pitch, gesture) opens up new possibilities for action (for herself and the other teammates). Mirna's embodied utterance is, indeed, a *call* to the Other.

3.1. Responsibility

The call now must be responded to, and it is responded to within a certain node of social relations that tie the students together. Whatever path the teammates' response takes, it is cast in a general ethical attribute that Lévinas calls *responsibility*. For Lévinas, responsibility is "the essential, primary, and fundamental structure of subjectivity … [where] the very node of the subjective is knotted" (1982, p. 101). Since all educational theories put into motion a certain ethics—for ethics is the substrate and form of our relation to the Other—responsibility is a common denominator of all of them. Yet, the *meaning* of responsibility is not the same. In the theory of direct instruction, the theory dictates that responsibility lies with the student to assume the submissive role vis-à-vis the teacher. In the case of the theory of objectification, responsibility means living and acting *with* and *for* others; it means to respond to the call of others as they are on their own terms: in their "existence, in [their] being-for-other[s] . . . as free being[s]" (Hegel, 1978, p. 57).

3.2. Commitment

Commitment is both a promise and its realization of doing everything possible to work side by side with others in the course of our joint labour (e.g., trying to understand the process being followed to solve a problem, trying to contribute to the classroom common work).

3.3. Care for others

Far from being an act of condescension or a patriarchal act, or simply caring for someone, the care for others is a pre-conceptual relational involvement entailing the attention and recognition of others and their material and spiritual needs. Although caring for the Other opens up the possibility of seeing ourselves in the Other, of recognizing our vulnerability in the vulnerability of the Other, the importance of caring for the Other is to go beyond ourselves, to be dragged powerfully into the world and to position ourselves there, with-the-Other.

To understand Mirna's "Ahhhhh!!!" within an ethical practice of responsibility, commitment, and care, we need to broaden our conception of language and *recognize* (in the Hegelian sense) this painful expression as *voice*; that is, as something where, as Lévinas suggests, the *saying* moves beyond the totalizing enclosure of the *said* and becomes rather the possibility of openness to the other (Radford, 2021a). In this conception of voice, power does not disappear since power is not a *thing*, but something imbricated in our

relations to others. What we can expect in the transformative movement towards a communitarian ethics is that, through conscious, reflective, and critical stance, power in the classroom goes beyond its own subjecting mechanisms of social order and becomes rather something fluid, dynamic, to be exercised with responsibility *for* the Other.

The communitarian ethics sketched here orients our pedagogical acts in the classroom, where teachers and students explore together new critical spaces that promote engagement, inclusiveness, debate, and respect (Radford, 2021b). This ethics is consonant with the conceptual bases of the theory of objectification and its conception of learning as a collective process. The communitarian ethics tries to reflect and operationalize the theory's conceptual stance in the kind of relationships between self and other. Underneath the communitarian ethics lies the recognition that our historical, cultural, and material origin embeds and refracts dynamic and antagonistic visions and conceptions of the world and of what a good life can mean. It is the vitality of contradictions that gives substance to social human life, always changing, always challenged, and that makes meaning a polyphonic event, always arising and evolving "in the context of struggle" (Juzwik, 2004, p. 540).

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